THE INTERNAL RATE OF RETURN PROBLEMS AND MANNERS OF SOLUTION

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Abstract

The Internal Rate of Return (IRR) as an evaluation criterion of investment projects was used and still is being used widely. However, it presents three disadvantages: a) the disadvantage of reinvesting the intermediate revenue, b) the late costs and c) the existence of many roots during solving out the respective mathematical equation. Therefore, to avoid jumping into misleading results-conclusions it is advisable to use this criterion carefully and on the same time to proceed to the required interferences-corrections when it is considered necessary.

Résumé

«Le taux d' intérêt interne» comme critére dévaluation de plans d' investissements a été utilisé et il est encore largement utilisé. Cependant il present trois problémes essentiels: le probléme de re-investissement des revenus intermediaires, le probléme de depenses retartaires et le probléme de l' existence de plusieurs racines derivantes de la solution de l' équation mathématique. Il est allors opportun d' utiliser ce critére avec prudence et d' effectuer les intervations – rejustements – la ou il est necessaire — afin de ne pas se mener vers de conclusions-resultats qui peuvent tromper.

Because of the limited available production factors, the fair distribution of the national resources and the need for maintaining the renewable natural resources in order to be used also by the future generations, the methods of investment evaluation and the respective discount rate used, hold andoubtedly a dominant position.
According to poet Elytis definition of environment – «Environment, as someone may realize it, is not a whole of land, plants and waters. It is the projection of people's soul upon the matter (Elytis, 1990) – the people's soul may be "x-rayed" via the assessment of the condition in which the natural environment is found in which people lives and creates. Therefore, it is possible, through the scientifically documented application of investment evaluation criteria and appropriate discount rate for someone who is responsible at a level of decision making about issues of environment conservation to hope that future generations will not curse their ancestors ...»
To evaluate the various investment projects three criteria are mostly used which take into account the intertemporal value of money (Marglin, 1967 - Watt, 1973 - Mishan, 1975 - Christodoulou, 1989):
a) the criterion of Net Present Value (NPV)
b) the criterion of Internal Rate of Return (IRR) and
c) the criterion of Benefit-Cost Ratio (B/C)
The application of these three criteria is based on the analysis of the same economic data. First, estimations of the net periodical revenues of every investment are required as well as determination of the discount rate. The discount rate, in the first and third criterion, is used for discounting the net periodical revenues whereas in the second criterion is used as comparison measure with the rate which the investment is expected to generate (IRR).

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In Yale, Chapman (1915) (according to Harou, 1985) introduces the concept of internal rate of return while Hilley (1930) (according to Harou, 1985), in Britain, shows how someone can calculate the internal rate of return based on Faustmann's formula.

Internal rate of return (IRR)

Internal rate of return is the discount rate which reduces the net present value of an investment project exactly to zero (Ministry of Overseas Development, 1977) or internal rate of return is the rate

of interest which makes the discounted revenues equal to the discounted costs (Price, 1989). Damalas (1979) for a timber production firm defines as internal rate of return the average of interest rate obtained over the entire costs made before the final harvest.

The internal rate of return criterion estimates the real interest rate which the investment generates and has the advantage that it does not pre-requires a knowledge of the discount rate, that is during its estimation no market's interest rate or time preference rate is taken into account.

Virtually, the internal rate of return represents the highest interest rate which an investor could pay without loosing money if he borrows the entire capital for the investment's funding and pays off the loan (initial amount and interests) with the revenues coming out from the investment paying at the moment they are made.

Marty (1970) supports that if the needed for the investment capitals can be borrowed with an interest rate smaller than internal rate of return or they can be withdrawn from other investments which yield a smaller rate than the internal rate of return then the financial position of the firm would be improved by carrying out the under consideration investment.

An individual investment becomes acceptable if the internal rate of return is bigger than a desirable interest rate which is usually the rate prevailing in the market.

Whether there are compatible investments, then these are graded in a decreasing order of size on the respective internal rates of return.

Last, in case of mutually incompatible investments the one with the higher internal rate of return, is chosen.

Problems of internal rate of return

The IRR is unquestionably used more by the responsinble analysts of various firms and by foresters as well. The main reason is that no calculation of the discount rate is required beforedhand (Webster, 1965 - Schallau et al., 1980). Yet, Foster et al. (1983) believe that IRR should become a typical analytical tool of forest investment evaluation.

However, despite its wide use the IRR is characterized from severe problems as well (Price, 1989 and 1993): the prerequisites of re-investing the intermediate revenues, the problem of late costs, and the problem of the existence of many roots during solving the respective mathematical equation. These problems led Price (1989) to conclude that this criterion should not be used. For the same reasons, Damalas (1979) stresses that «uncontrolled usage of IRR for evaluating investment projects in forestry may lead to wrong decisions» and recommends to use it carefully in conjunction with also other criteria (net present value and benefit - cost ratio).

The multiple roots problem

According to the definition of the IRR for somebody to be able to estimate its precise height, as long as we refer to a specific investment project, we should solve the equation:

$$NPV = \sum_{t=0}^{t=T} \frac{R_t}{(1+i)^t} - \sum_{t=0}^{t=T} \frac{C_t}{(1+i)^t} \qquad (1)$$

where:

R_t, C_t = the revenues and costs respectively, per year
T = the investment lifetime
i = the discount rate

However, many equations have more than one solutions. That happens when revenues and costs interchange intertemporally (Marty, 1970 - Price, 1993). If this is the case then what solution should be adopted? Suppose for example, that a timber-trading company offers one million drs. to exploit the wood of a forest section. Three years later, when felling is completed, the Forest Service establishes a plantation of fast-growing species which costs 2,5 million drs.; after 12 years from its establishment, the plantation is harvested and provides net revenues of five million drs. To calculate, in this case, the IRR we solve the equation:

$$1.000.000 - \frac{2.500.000}{(1+IRR)^3} + \frac{5.000.000}{(1+IRR)^{15}} = 0$$

from which we calculate two IRRs, one equal to 14,28% and a second one equal to 32,58%.

Therefore, what would be the real IRR? In fact, both IRRs are real because both make the discounted revenues equal to the discounted costs. But Hirshleifer (1958) points out that IRR is interpreted as a development rate and the investment, naturally, can not be developed concurrently with two IRRs. Wright (1963) quotes regulations under which the lower IRR coming out from positive/negative/positive revenues is considered as the authentic one. The highest IRR is simply a lending/borrowing rate for which the investment would be located at «its break even point» (in other words it would have had neither profit nor damage). Marty (1970), believes that many times there are two or more IRRs because the facts of the investment are not fully defined. That is, usually nothing is said about how the intermediate collected revenues are going to be used; whether they will be re-invested or not and with what exactly interest rate. Therefore, the cause for having two IRRs for some investment projects is the fact that there are two re-investment interest rates which when apply for the re-investment of intermediate revenues each one of them will respectively bring on an equal IRR.

According to the above stated let us assume we have an investment project which brings out a cost of 10 million drs. in year zero, revenues of 50 million drs. in 10th year and a cost of 60 million drs. in the 20th year. For determining the IRR we must solve the equation:

$$10.000.000 + \frac{50.000.000}{(1+IRR)^{10}} - \frac{60.000.000}{(1+IRR)^{20}} = 0$$

from which we find two IRRs equal to 7,25% and 11,25%.

Now, if the intermediate revenues of 50 million drs. will be re-invested until the 20th year with an interest rate 7,25% their value at the end of that year will equal to 50.000.000 X $(1+i)^{10}$ = 100.680.000 drs. If we take off the 60 million investment taking place at the 20th year, we will have a net output equal to 40.680.000 drs. That is the 40.680.000 drs. is the outcome of the 10 millions drs. invested 20 years ago; therefore, we have:

$$10.000.000 \times (1+IRR)^{20} = 40.680.000$$

from which it comes that IRR=7,25%.

Certainly, the same will happen if we apply as re-investment rate of intermediate revenues the second value of the IRR, that is 11,25%.

In contrast to the problem of multiple roots there is also the case of cash flow of an investment which do not show any IRR. For example, we assume we have the following cash flow: +10 mil drs. in year zero,20 mil drs. during first year and +40 mil drs during the second year. For such a case, obviously, it is not possible to apply the IRR criterion.

The late costs problem

Costs which occur during the last years of an investment project is possible to lead to irrational conclusions. For example, we consider a forest exploitation project which produces net revenues of 1 million drs. at the end of every year for ten continuously years. At the end of the 10th year the forest is completely destroyed due to devastating floods and soil's erosion, causing a damage estimated of one billion. The IRR of this unusual incident is defined by solving the equation:

$$\frac{1.000.000}{(1+IRR)^1}+\frac{1.000.000}{(1+IRR)^2}+\ldots+\frac{1.000.000}{(1+IRR)^{10}}-\frac{1.000.000.000}{(1+IRR)^{10}}=0$$

from which we find IRR = 99,4% !!!
The explanation of this unusual result is based on the fact that in order to confirm the above equation the big future costs should be «discounted heavily» something that may be done by using big discount rate. Indeed, if the damage scale was even bigger we should have used even bigger discount rate which would mean that we could find bigger IRR; in other words the project would appear more profitable!!
To the claim that problems created by the late costs or by the multiple roots of IRR are nothing else but «fabrications»î of non-realistic examples (Foster et al., 1983), Price (1989 and 1993) replies that late and long-standing environmental and social costs - such as floods, greenhause effect, loss of genetic resources, maintainance expences ets. - appear to be fairly characteristic examples for big development projects.
A combination of multiple IRRs and late costs deepens the confusion. Let us, for example, have the projects I and II of the **table 1**, with the respective cash flow. To calculate e.g. the IRR of the first project we must solve the equation:

Table 1 ***Cash flow of projects I and II.***

Time	Project I	Project II
0	–1.000.000	–1.000.000
1	2.000.000	2.000.000
2, 3, 4, ... ∞	–100.000	–200.000
Low IRR	11,3%	27,6%
High IRR	88,7%	72,4%

$$-1.000.000+\frac{2.000.000}{(1+\text{IRR})^1}-\frac{\frac{100.000}{\text{IRR}}}{(1+\text{IRR})^2}=0$$

from which we find two values for the IRR equal to 11,3% and 88,7%.
For project II the values of IRR are 27,6% and 72,4%. According to the above table the project II has double late costs (200.000 drs.) in comparison to project I (100.000 drs.). Consequently, while by common sense we choose project I (since all other cash flow are identical), the IRR test is ambiguous; that is if we use smaller IRRs we choose the project II while by using higher ones we choose the project I.
The higher IRR is the maximum interest rate up to which the investment can «bear» borrowing money for the initial funding of the project (Wright, 1963). Therefore, project I faces an easier direct problem, since it can borrow money until 88,7% for covering the initial cost. Howerer, futherdown, the 88,7% becomes the minimum interest rate which the Heads of project must be in position to lend money in order to collect capitals to cope with the costs occuring later on; so, project II is the one which has to face the easier problem.
Therefore, all these peculiar results come out as a consequence of the fact that for the estimation of IRR we virtually solve an equation: discounted positive cash flow must be equal to the discounted negative ones. Another natural consequence of this fact is that if the signs of a project cash flow are reversed then the IRR will be exactly the same. For example, the IRR of a project with cost of 200 drs. in the year zero and revenues 2.000 drs. in the tenth year is equal to 25,9%. But the same exactly IRR will come out if we had revenues 2.000 drs. in the year zero and cost 200 drs. in the tenth year. Therefore, an investment evaluation criterion which may be ìindifferentî whether the items of cash flow are costs or revenues, it can not but create suspicions about its efficiency (-Price, 1993).

The intermediate revenues re-investment problem

The IRR itself as a solution of some mathematic equation does not contain any prerequisite in respect to the re-investment of intermediate revenues.
However, the IRR may only be interpreted as a long-term growth rate if re-investment does take place in projects of the same profitability (Price, 1993). That manner by which the IRR of an investment is estimated is a process indicating that we have a compound interest rate (Marty, 1970).
However, practically, there may be not adequate re-investment probabilities, a fact which creates some questions about the real height of IRR. For this, Schallau et al. (1980) believe that it has not been payed after all the proper attention on the relation existing between the re-investment interst rate and re-investment possibility of intermediate revenues.
We take, as an example, an investment project with initial cost 30.000 drs. which leads to annual net revenues of 24.000 drs. for 90 years. Ih this situation IRR is given by solving the equation:

$$-30.000+\frac{24.000}{(1+\text{IRR})^1}+\ldots+\frac{24.000}{(1+\text{IRR})^{20}}=0$$

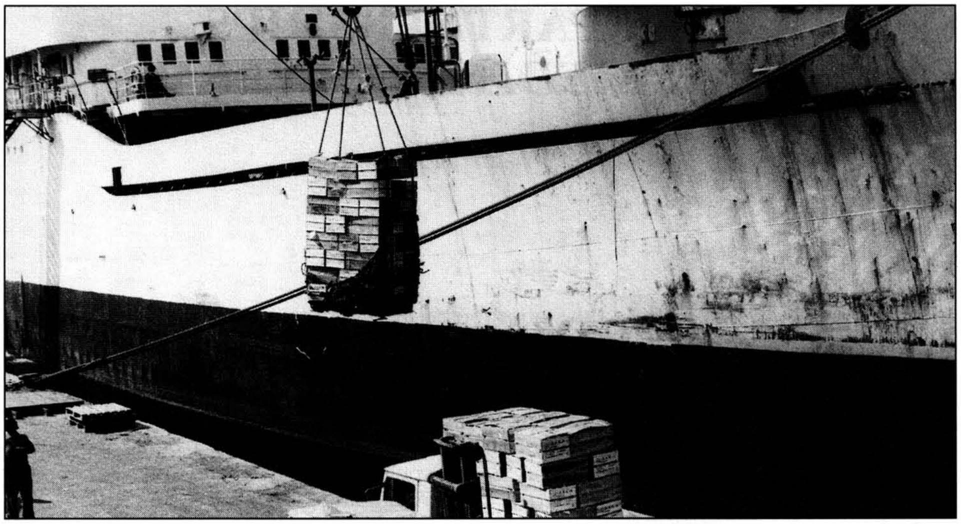
or by using the synoptical mathematical formula:

$$\text{C}=\frac{\text{R}\left[(1+\text{IRR})^{\text{T}}-1\right]}{\text{IRR}(1+\text{IRR})^{\text{T}}} \quad (2)$$

where:
C = the initial investment cost
R = the annual net revenues
T = the investment lifetime
we have:

$$30.000=\frac{24.000\left[(1+\text{IRR})^{90}-1\right]}{\text{IRR}(1+\text{IRR})^{90}}$$

from which we find IRR=80%.
However, is that rate a realizable rate of return (RRR)? Certainly it is, but only in the case the businessman can re-invest the annual net revenues with the same interest rate, that is 80%. But, if the re-investment's interest rate is smaller, then the Head in-charge should know that the IRR which was found will be virtually misleading. Therefore, in order to have correct results the following procedure is recommended (Marty, 1970 - Schallau et al., 1980): if we assume that the re-



investment rate is i, then we find the final capitalized value (that is the value at the end of investment lifetime T) of equal net revenues and we equate it with the product C $(1+RRR)^T$, that is:

$$C(1+RRR)^T = \frac{R\left[(1+i)^T - 1\right]}{i} \quad \text{or}$$

$$RRR = \left(\sqrt[T]{\frac{R\left[1+i)^T - 1\right]}{iC}} - 1\right)100 \quad (3)$$

where:
RRR = the realizable rate of return
i = the re-investment rate
C = the initial investment cost
T = the investment lifetime

If, for the above example, we assume that the re-investment interest rate is equal to 10%, we will have:

$$RRR = \left(\sqrt[90]{\frac{24.000(1,1^{90} - 1)}{0,1 \times 30.000}} - 1\right)100 \quad \text{or}$$

$$RRR = 12,56\%$$

Consequently, if the reinvestment interest rate is only 10% then the RRR of investment will be 12,56%. Naturally, someone can try various re-investment rates of return finding also various RRRs. Marty (1970) defines the RRR of Schallau et al. as composite internal rate of return (CIRR) and provides the following generalized methamatical formula for his calculation:

$$(1+CIRR)^T \sum_{j=0}^{j=T} \left[C_j / (1+i)^j\right] = \sum_{j=0}^{j=T} \left[R_j (1+i)^{T-j}\right] \quad (4)$$

where:
CIRR = the composite internal rate of return
Cj = the costs occuring in year j
Rj = the revenues occuring in year j
i = the reinvestment rate
T = the investment lifetime

According to formula (4), Marty having determined a rate of reinvestment of intermediate revenues (including the capital cost which is maybe necessary during the operation of investment) he calculates a respective initial equivalent cost and a final equivalent revenue. Therefore, he calculates only one value for CIRR from the final formula:

$$CIRR =, \left(\sqrt[T]{\frac{\sum_{j=0}^{j=T}\left[R_j (1+i)^{T-j}\right]}{\sum_{j=0}^{j=T}\left[C_j / (1+i)^j\right]}} - 1\right)100 \quad (5)$$

The CIRR comes into complete accordance with the criterion of net present value (NPV).
The advantage of CIRR will not cease to exist even in the case that the investment evaluation is more complicate e.g. an investment which also has intermediate costs and intermediate revenues while the interest rates by which the firm borrows or lends money also differ. Then, of course, we will need to discount intermediate costs with the interest rate by which the firm borrows money, and reinvest intermediate revenues with the respective interest rate by which the firm lends money (reinvestment interest rate).
Moreover, even these interest rates is possible to change in the lifetime of an investment.

Conclusions

The IRR constitutes an evaluation criterion of investment projects used widely since it does not require a knowledge on discount rate. Howerer, it presents three basic problems:

1. The problem of multiple roots which very offen come out from the solution of the respective mathematical equation. The lowest IRR is considered as the authentic one whereas the highest IRR is simply a lending/borrowing rate for which the investment is found at «the break even point» (in other words it will not have profit nor damage).

2. The problem created by late costs. In fact, the costs realized in the far future from the beginning of the investment's operation is possible to lead to misleading results-conclusions and this should be particularly taken into consideration.

3. The problem of re-investing the intermediate revenues. The IRR can only interpreted as a long-term growth rate if re-investment does take place in projects of the same profitability. In the opposite situation we must calculate the real reinvestment rate of intermediate revenues and determine the real IRR, respectively. ●

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