

# Maximization of the expected return-profit of a multi-productive enterprise under conditions of uncertainty – Practical application in a food firm

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## Introduction

The goal of any company producing a variety of products is to achieve the highest possible profit that originates from the contribution of each product, as regards the percentage of its sales in relation to the other company products.

In the competitive world of business, companies try to ensure that the mix of products they produce and sell is such to avoid keeping excess stock but also to satisfy the level of demand, since extensive product storage incurs additional related costs and the lack of demand satisfaction causes the revenue loss.

The company is therefore called upon to decide the quantities to produce, within the framework of its capacity on the one hand and its available resources on the other hand, in order to avoid the additional cost of storage and the revenue loss, while simultaneously achieving the maximum possible profit. This means that it shall ensure that its product mix or portfolio is able to provide the maximum possible return, as in the case of stock portfolios.

By taking certain parameters into account in this paper, we try to arrive at a mathematical formula that is easy to comprehend and apply, so that it may be a useful decision-

## Abstract

The aim of the present paper is to determine the optimum level of production for each product a company produces so that, by knowing the probability density function of the demand for the above-mentioned products, we can achieve the maximum possible results within the framework of the company's capacity on the one hand and of the available resources on the other hand. In the present paper, we close with a formula correlating the probability density function of the demand with a given specific production presenting the selling price and the variable costs both subjected to changes. By collecting data related to the sale of each product, we formulate or arrive at the form of the demand distribution through the method of the "adaptation to the curve", which corresponds to different production levels for each product. Then, by taking the sum of the maximum return-profit for each product that corresponds to a specific production, we arrive at the desired result. We believe that this particular methodology, in combination with the principles of financial reporting and costing, has much to offer given that the use of this model constitutes a competent tool through which the management team may organize its product portfolio in order to achieve the maximum possible results.

**Keywords:** optimization, uncertainty, food sector.

## Résumé

*L'objectif du présent travail est de déterminer le niveau de production optimal pour chaque produit réalisé par une entreprise de manière à obtenir le maximum des résultats possibles sur le plan de la capacité de l'entreprise et des ressources disponibles, en s'appuyant sur la fonction densité de probabilité de la demande de ces mêmes produits. Nous sommes parvenus à une formule qui met en corrélation la fonction densité de probabilité de la demande et une production spécifique, en réunissant le prix de vente et les coûts variables, assujettis tous les deux à des changements. En collectant les données relatives à la vente de chaque produit, nous pouvons formuler ou déterminer la forme de la distribution de la demande à travers la méthode de l'« Adaptation à la courbe », qui correspond à différents niveaux de production pour chaque produit. Successivement, en considérant la somme du rendement-profit maximum pour chaque produit correspondant à une production spécifique, nous obtenons enfin le résultat escompté. Nous estimons, donc, que cette méthodologie particulière, associée au rapport et au coût financiers, est très prometteuse étant donné que ce modèle constitue un outil pertinent pour les gestionnaires, appelés à organiser leur gamme de produits afin d'atteindre le maximum des résultats possibles.*

**Mots clés:** optimisation, incertitude, secteur alimentaire.

making tool for the company management. The resulting model is stochastic but also general, in the sense that it could refer to any multi-productive enterprise. The application below refers to one sector and involves a Greek dairy industry.

As defined above, one significant factor is the production cost and another is the pricing of each product, both dictating whether a product should be withdrawn or whether its demand should be covered in various other ways.

All this influences the fundamental issue of cost accounting for the various products and whether product prices should or should not be dependent on costing methods. The role of cost in decision-making has been the subject of extensive debates in managerial accounting (Thomas, 1974). Pricing is after all part of the definition problem: a) which products should we produce and which not? b) what capacity should we distribute the available re-

sources?

Since traditional costing systems do indeed have constraints, many researchers of managerial accounting have made efforts to reduce these constraints by focusing on costing systems (Activity Based Costing). Costing systems have emerged as a solution to competitiveness and product mix decision-making (Kaplan, 1994; Cooper, 1988).

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Despite the fact that great attention has been paid to the ABC systems, there are claims that they do not respond to strategic demands (Foster and Gupta, 1994; Karmarker *et al.*, 1994). While their benefits are considered substantial on a theoretical level, in practice they have not lived up to expectations, since they have not been sufficiently applied so far (Innes and Mitchell, 1995; Chenhall and Langfield – Smith 1998; Anderson and Young, 1999).

Several people believe that the value of ABC systems has been overestimated (Piper and Walley, 1990; Morgan and Bork, 1993), whereas others state that ABC systems should be redefined (Johnson, 1988) and question their ability to determine precise costs when they are undeclared (Datar and Gupta, 1994; Maher and Marais, 1998). Others examine the ABC systems before and after their acceptance (Murphy and Braund, 1990).

From the above-mentioned data, the question that arises is whether we should use the ABC systems or the Full-Cost System, in order to determine the pricing policy (Gox, 2001, 2002). Gox proposes that any applied model should initially refer to one period, due to the elasticity of demand, which is a view wholeheartedly embraced by the present author, given that results-profits in Financial Reporting and Cost Accounting are measured for one fiscal period. Furthermore, Gox is also a supporter of the Full-Cost System, which we also accept and adopt.

In this paper, we consider that the raw materials used, the direct labour and the General Industrial expenses determine the variable cost. I will not expand on the method of allocation of General Industrial expenses, since that is not the purpose of this paper. In any case, the writer's opinion (Gox 2001, 2002) has been accepted to a varying degree by other authors, such as Banker and Hansen (2002), Balachadran *et al.*, (2002), BalaKrishnan (2001).

It is also important to mention the contribution of Balakrishnan *et al.* in designing various product mixes in order to arrive at the maximum possible result.

Of course, they also agree that conclusions resulting from their proposed models do not lead to reliable results, given that they do not stochastically determine the demand and do not even relate it to various production levels per product, as targeted by this paper. They simply refer to the grand model in stochastic demand in a general way, without going too much into details about approaches and definitions.

Following the introductory section and the review of the relevant literature, the present paper is structured as mentioned below.

The second part – section of the paper deals with the problem of the products' synthesis, as well as the conditions, assumptions or constraints that must be taken into account for the company to achieve the maximum possible result. In this part, we also develop the methodology used to estimate demand, which is precisely the reason why the present paper should prove to be useful and mainly applicable for any company.

In the third section, the method is practically applied on a classical Greek dairy company and its relevant collected and processed data and refers to a long period of time.

The fourth section includes an analysis of the conclusions from the proposed model and the afore-mentioned application, along with some proposals and further thoughts on the subject, as we certainly do not claim that the recommended model can resolve all problems, particularly in view of the fact that we live in an uncertain and extremely competitive environment.

## 2. Profit maximization under conditions of uncertainty involving the production of numerous products.

Given that the company in question produces and sells numerous products within a fully competitive environment, two problems automatically emerge. The first concerns the allocation of available resources for product production in combination with the company's capacity. The second involves determining the demand for each product.

The aim of any company is to link the available resources with the demand for each product so that each product's contribution brings about the maximum possible result. The selected product portfolio must ensure that: a) we avoid high levels of stock, which require the commitment of available resources that could potentially have been used for some other products whose demand was not met, b) we avoid expenses related to product maintenance and storage and c) we avoid expiry of such products.

The following problem then arises: to estimate the demand for each product that corresponds to a certain level of production. More specifically: the production of each product corresponds to a specific demand, obviously with a degree of probability. We must begin from the estimation of this demand, which is initially unknown.

The method used in this document for the definition and estimation of the probability distribution of demand, in case it does not constitute one of the known theoretical distributions, is the "adaptation to the curve with the use of historical data" method.

The adaptation to the curve consists of the following:

If we have a series of observations  $x_i$ , ( $i = 1, 2, \dots, n$ ),  $f(\alpha, \beta, \dots, \kappa)$  is the distribution for adaptation,  $\mu = m$  ( $\alpha, \beta, \dots, \kappa$ ) is the mean of this distribution and  $\bar{x}$  is the mean of our observations  $x_i$ , then we can calculate the parameters  $\alpha, \beta, \dots, \kappa$  from the system of equations

$$\bar{x} = m.(\alpha, \beta, \dots, \kappa)$$

$$\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^r = \int_{-\infty}^{\infty} (x - m)^r f(\alpha, \beta, \dots, \kappa) dx, \quad r=2, 3, \dots, \kappa$$

and continue by taking as many moment equations as the number of parameters to be estimated.

More specifically:

Either from past experience or from the form of the data distribution histogram, we know that our random variable follows some pattern of distribution, but we are not aware of the parameters that define it.

From the observations available, we estimate the parameters of this distribution.

Then, through the moment equation method (curve adaptation) and provided we know the expression of each theoretical moment in conjunction with the parameters, we solve the system of equations that arises.

Therefore, by knowing the theoretical moments of distribution  $EX^r = \varphi_r(\alpha, \beta, \dots, \kappa)$ ,  $r=1, 2, \dots, \kappa$ , we estimate the

parameters from the solution of the  $EX^r = \frac{1}{n} \sum x_i^r$   $r = 1,$

$2, \dots, \kappa$ , system and proceed with the solution of the problem we are faced with, i.e. the stochastic determination of demand.

Depending on which of the variables that compose the company's revenue or expenses are random or not, we attempt at defining the value of the expected revenue  $E$  (TR) and the expected expenses  $E$  (TC), in order to determine the maximum profit.

In this way, we manage to ascertain the probability density function of a product's demand according to the quantity produced. The same process should then be repeated for all the company's products based on the available data. We certainly do not claim that this is a simple process. On the contrary, it is quite complex and often time-consuming. However, we believe that if it is applied for one fiscal period, then it will constitute a full guide for improving the proposed model and avoiding unnecessary levels of production.

Our next step is to define the objective function of the maximum profit for each product under the conditions and constraints that we impose. If we take the sum of the maximum profit for each product, we will then, in our opinion, arrive at the final maximization of the total profit of the said company.

More specifically:

This whole paper is based or otherwise founded on the elasticity of demand that is related and refers to a specific level of production, and in particular:

a) On the historical data of a multi-productive company regarding selling prices, variable costs and the percentage of each product's sales in relation to total company sales, which lead us to the mean contribution margin for each product on the one hand and to the average sales on the other hand.

b) The production cost of each product consists of the cost in raw materials, the direct labour and the General industrial expenses.

c) We determine the demand probabilities that correspond to a particular production level, i.e. which probabilities there are that the demand will be equal to or lower than the production.

d) In this way we arrive at the particular demand for each product or at the specific production level corresponding to each product.

e) Based on the above-mentioned points and taking account certain constraints, such as the available resources, the company's capacity and the existence of a positive contribution, we arrive at the objective profit function for one product.

f) Finally, after repeating the same process for all products, we take the sum of the maximum profits of all products.

Next, according to the results, we assume that the selling price  $P_1$  of product  $i$ , where  $i = 1 \dots n$ , is initially determined by the Full-cost method and that we know it. The variable cost  $V_1$  of product  $i$ , where  $i = 1 \dots n$ , is also known.

The demand probability  $P_1 (Q_{s1}/Q\pi_1)$  is known and the production of each product is given and, therefore, our objective is the profit maximization, where refers to the product demand and to the production of a particular product.

We then come to the following formulae for the expected revenue and expenses. This difference will obviously provide us with the maximum expected profit. Since we will be using discontinuous variables in this paper, the maximization of the expected profit function will be found by applying the calculus of determinate differences. The first and second difference will have to be found in correspondence with the first and second derivatives used in the continuous variables. The first and second difference are defined by the following formulae respectively:

However, in this specific case, we are not interested in the local maximum but in the general maximum. There-

$$\Delta f(x) = f(x) - f(x-1) \quad (2.1.)$$

$$\text{and } \Delta^{\Gamma+1} f(x) = \Delta(\Delta^{\Gamma} f(x)) \text{ or } \Delta^{\Gamma} f(x) = \Delta(\Delta^{\Gamma-1} f(x)) \quad (2.2.)$$

fore, in order to have an absolute maximum for the expected profit function  $E (TR_1/Q\pi_1)$ , the following conditions should be met:

According to the above data, the expected revenue for the one product  $E (TR_1/ Q\pi_1)$  will be:

$$\Delta E(\pi_o / Q\pi) \geq E(\pi / Q\pi) \quad (2.3.)$$

$$\text{and } \Delta^2 E(\Pi_o / Q\pi) \leq 0 \quad (2.4.)$$

$$(2.5.)$$

$$E(TR_1 / Q\pi_1) = \sum_0^{Q\pi_1} [(P_1 Q_s) - P(Q\pi_1 - Q_{s1})] P_1(Q_{s1} / Q\pi_1) + \sum_{Q\pi_1+1}^{\infty} [(P_1 Q\pi_1) - P_1(Q_{s1} - Q\pi_1)] P_1(Q_{s1} / Q\pi_1) \quad (2,5.)$$

with  $Q_{s1} < Q\pi_1$

and with  $Q_{s1} \geq Q\pi_1$ .

Similarly, the expected expenses for the one product E ( $TC_1 / Q\pi_1$ ) will be:

$$E(TC_1 / Q\pi_1) = \sum_0^{Q\pi_1} [(V_1 Qs_1) - V_1(Q\pi_1 - Qs_1)] P_1(Qs_1 / Q\pi_1) + \sum_{Q\pi_1+1}^{\infty} [(V_1 Q\pi_1)] P_1(Qs_1 / Q\pi_1) + FC_1 \quad (2.6.)$$

with  $Qs_1 < Q\pi_1$

and with  $Qs_1 \geq Q\pi_1$ .

If we deduct formula (2.6.) from (2.5.), we arrive at the expected profit for the product.

$$E_1(\Pi_1 / Q\pi_1) = \sum_0^{Q\pi_1} [P_1 Qs_1 - P_1(Q\pi_1 - Qs_1) - V_1 Qs_1 + V_1(Q\pi_1 - Qs_1)] P_1(Qs_1 / Q\pi_1) + \sum_{Q\pi_1+1}^{\infty} [(P_1 Q\pi_1) - P_1(Qs_1 - Q\pi_1) - V_1 Q\pi_1] P_1(Qs_1 / Q\pi_1) - FC_1 \quad (2.7.)$$

In our opinion, the above-mentioned formula can be simplified in order to facilitate its application. Thus, formula (2.7) can acquire the following form:

$$E_1(\Pi_1 / Q\pi_1) = \left[ (P_1 - V_1) \sum_0^{Q\pi_1} Qs_1 P_1(Qs_1 / Q\pi_1) - P_1 \sum_0^{Q\pi_1} (Q\pi_1 - Qs_1) P_1(Qs_1 / Q\pi_1) + V_1 \sum_0^{Q\pi_1} (Q\pi_1 - Qs_1) P_1(Qs_1 / Q\pi_1) + Q\pi_1 (P_1 - V_1) \sum_{Q\pi_1+1}^{\infty} P_1(Qs_1 / Q\pi_1) - P_1 \sum_{Q\pi_1+1}^{\infty} (Qs_1 - Q\pi_1) P_1(Qs_1 / Q\pi_1) - FC_1 \right] \quad (2.8)$$

where:

a)  $P_1(Q\pi_1 - Qs_1)$  is the loss of revenue due to excess production;

b)  $P_1(Qs_1 - Q\pi_1)$  is the loss of revenue due to excess demand;

c)  $V_1(Q\pi_1 - Qs_1)$  is the end-of-year stock according to the principles of financial reporting;

d)  $FC_1$  = fixed expenses allocated to product  $Q\pi_1$ ;

e)  $\Pi_1(\Pi_1 / Q\pi_1)$  is the total expected return from product  $Q\pi_1$  with a given-specific level of production.

Under the following terms and conditions:

a) the total produced quantity of each product should exceed neither the company's productive capacity nor its available resources.

$$\text{i.e. } \sum_{i=1}^N P_i Q\pi_i \leq Lij \text{ or } \sum_{i=1}^N P_i Q_i - Lij \leq 0$$

where  $Lij$  are the available resources.

b) the contribution of each product should be positive i.e.  $P_i - V_i > 0$

c)  $\sum_{i=1}^n Q\pi_i \leq Q_n$  where  $Q_n$  is the company's maximum capacity.

d) the elasticity of the demand should be greater than zero  $\Delta > 0$ .

e) the demand for product  $i$  should not be affected by or affect the demand for other products and vice versa, e.g. the demand for feta cheese must not depend on the demand for yoghurt and vice versa.

From formula (2.8), we get the first difference regarding  $Q\pi_1$ , which is:

$$\Delta E_1(\pi_1 / Q\pi_1) = (V_1 - P_1) \sum_0^{Q\pi_1} P_1(Qs_1 / Q\pi_1) + (2P_1 - V_1 + 1) \sum_{Q\pi_1+1}^{\infty} P_1(Qs_1 / Q\pi_1) \quad (2.9)$$

$$\text{or } \Delta E_1(\pi_1 / Q\pi_1) = (V_1 - P_1) \sum_0^{Q\pi_1} P_1(Qs_1 / Q\pi_1) + (2P_1 - V_1 + 1)$$

$$\left[ 1 - \sum_0^{Q\pi_1} P_1(Qs_1 / Q\pi_1) \right] \quad (2.10)$$

since  $\sum_{Q\pi_1+1}^{\infty} P_1(Qs_1 / Q\pi_1)$  is equal to formula

$$\sum_0^{Q\pi_1} P_1(Qs_1 / Q\pi_1) - \sum_0^{Q\pi_1} P_1(Qs_1 / Q\pi_1) + \sum_{Q\pi_1+1}^{\infty} P_1(Qs_1 / Q\pi_1).$$

If we transform formula (2.10), we have

$$\Delta E_1(\pi_1 / Q\pi_1) = (V_1 - P_1) \sum_0^{Q\pi_1} P_1(Qs_1 / Q\pi_1) + (2P_1 - V_1 + 1) - (2P_1 - V_1 + 1) \sum_0^{Q\pi_1} P_1(Qs_1 / Q\pi_1)$$

$$\text{or } = (V_1 - P_1 - 2P_1 + V_1 - 1) \sum_0^{Q\pi_1} P_1(Qs_1 / Q\pi_1) \leq -(2P_1 - V_1 + 1)$$

and through  $\sum_0^{Q\pi_1} P_1(Qs_1 / Q\pi_1) = U_1(Qs_1)$  we finally arrive at

$$U_1(Qs_1) \leq \frac{2P_1 - V + 1}{3P - 2V + 1} \quad (2.11).$$

The above-mentioned formula (2.11) is indeed valid, since the first part of the inequality is the cumulative demand probability for a given  $Q\pi_1$ , while the second part of the inequality is smaller than the unit, which is a precondition of the cumulative probability.

We then create a table that contains the following data of interest:

- a) the values of  $Q\pi_i$  sequentially
- b) the probabilities of  $Qs_i$  with the corresponding values of the random variable  $Q\pi_1$
- c) the cumulative probability of  $Qs_i/Q\pi_i$
- d) by replacing the above-mentioned data and the values of  $P_i$  and  $V_i$  in formula (2.9), we arrive at the corresponding  $\alpha'$  differences.
- e) from the  $\alpha'$  differences column, we then find its second differences (2.9).

When the relevant table is complete, we then find the value of  $Q\pi_i$  so that formulae (2.3) and (2.4) are simultaneously valid.

Finally, formula (2.11) connects the probability density function of demand with known variables and shows the required production for each product that corresponds to a particular demand probability. Thus, we avoid random action and the excessive production and wastage of available resources it involves, while formula (2.8) is simultaneously used to determine the maximum profit for each product. The number of times that formula 2.8 is used will depend on the number of products we produce. If we then take the sum of the maximum profits for each product, we will manage to resolve the problem of profit maximization for the multi-productive company.

### 3. Application

We will examine the case of white cheese in a dairy industry that produces a whole range of products. The process we will follow for white cheese will also be repeated for all the other products. In the end, the sum of the individual profit functions will be added up, and will result in the total expected profit for the whole dairy industry in question.

The selling price and the variable cost per unit are determined for the period under examination, i.e.  $P = 100.00$  €/ton and  $V = 80.00$ €/ton.

The FC that corresponds to the specific product after the preceding distribution equals 29,000.00 €. The maximum production that can be realized for this specific product is 5,000 tons, while the minimum quantity that can be produced so that it is not crossed off the product portfolio is 500 tons.

After conducting a survey of data concerning this partic-

Table 1 – The probability of demand being lower or higher than production

$Q\pi_i$ values (thousands of tons)	Probability of $Qs$ (demand) being smaller or equal to $Q\pi$ (production)	Minimum expected quantity to be sold so that $Qs \geq Q\pi$
500	0.10	$500 \times 0.90 = 450$
1500	0.20	$1500 \times 0.80 = 1200$
2000	0.30	$2000 \times 0.70 = 1400$
2500	0.40	$2500 \times 0.60 = 1500$
3000	0.45	$3000 \times 0.55 = 1650$
3500	0.70	$3500 \times 0.30 = 1050$
4000	0.80	$4000 \times 0.20 = 800$
4500	0.85	$4500 \times 0.15 = 675$
5000	0.90	$5000 \times 0.10 = 500$

Table 2 – The relevant probabilities of the demand being equal to the relevant production (the observations are per 50 thousand tons)

Production $Q\pi = \alpha$ $0 \leq \alpha \leq 500$	Probability $\Pr(Q_s = / Q\pi = \alpha)$	Minimum expected quantity to be sold in thousands of tons
0	0.000 $P_0(Q_s=0/Q_\pi=0)$	0
50	0.000 $P_1(Q_s=0/Q_\pi=50)$	0
100	0.000 $P_2(Q_s=0/Q_\pi=100)$	0
150	0.000 $P_3(Q_s=0/Q_\pi=150)$	0
200	0.000 $P_4(Q_s=0/Q_\pi=200)$	0
250	0.020 $P_5(Q_s=5.00/Q_\pi=250)$	5
300	0.050 $P_6(Q_s=15.00/Q_\pi=300)$	15
350	0.150 $P_7(Q_s=52.50/Q_\pi=350)$	52.50
400	0.200 $P_8(Q_s=80.00/Q_\pi=400)$	80.00
450	0.250 $P_9(Q_s=112.50/Q_\pi=450)$	112.50
500	0.330 $P_{10}(Q_s=165.00/Q_\pi=500)$	165.00
	1.00	430.00 tons E ( $Qs$ )

ular dairy industry over the last five years, regarding selling prices, variable cost, quantities sold, levels of production and end-of-year stock, combined with the use of the probability determination method with the “adaptation to the curve”, we arrived at Tables 1 and 2.

From Table 2 we conclude that when the production is from 0 to 500 tons, the minimum expected quantity that is sold equals 430 tons.

Data in Table 3 result from repeating the process of Table 2 as regards the demand probabilities for white cheese up to a production level of 5000 thousand tons. In order not to mention the relevant analytical tables, we collectively present the results:

Table 3 – Minimum expected quantity to be sold according to the probabilities in Table 1.

Production ( $Q\pi$ )	Minimum expected quantity to be sold according to the probabilities in Table 1.
500	430 ( $P_1=10\%$ )
1500	1065 ( $P_2=20\%$ )
2000	1510 ( $P_3=30\%$ )
2500	1870 ( $P_4=40\%$ )
3000	2085 ( $P_5=45\%$ )
3500	2488 ( $P_6=70\%$ )
4000	2920 ( $P_7=80\%$ )
4500	3375 ( $P_8=85\%$ )
5000	3500 ( $P_9=90\%$ )

Based on data in Table 1 and on the values of P and V respectively, we create Table 4 with the  $\alpha'$ -  $\beta'$  differences, taking into account formula (2.9) as follows:

$$\Delta E(\pi_1 / Q\pi_1) = (V_1 - P_1) \sum_0^{Q\pi_1} P_1(Qs_1 / Q\pi) + (2P - V + 1) \sum_{Q\pi+1}^{\infty} P_1(Qs_1 / Q\pi_1)$$

that means we have

Table 4 –  $\alpha'$  and  $\beta'$  differences.

	$\alpha'$ difference	$\beta'$ difference
1. (500) tons = (80.00-100.00).010+(201.00-80.00).0.90	106.9	-
2. (1,500) «» = (80.00-100.00).0.20+(201.00-80.00).0.80	92.8	-14.1
3. (2,000) «» = (80.00-100.00).0.30+(201.00-80.00).0.70	78.70	-14.1
4. (2,500) «» = (80.00-100.00).0.40+(201.00-80.00).0.60	64.60	-14.1
5. (3,000) «» = (80.00-100.00).0.45+(201.00-80.00).0.55	57.55	-7.05
6. (3,500) «» = (80.00-100.00).0.70+(201.00-80.00).0.30	22.30	-35.25
7. (4,000) «» = (80.00-100.00).0.80+(201.00-80.00).0.20	8.2	-14.1
8. (4,500) «» = (80.00-100.00).0.85+(201.00-80.00).0.15	1.15	-7.05
9. (5,000) «» = (80.00-100.00).0.90+(201.00-80.00).0.10	-5.9	-7.05

We see that the sign for the  $\alpha'$  differences changes at 4,500 tons. This means that, theoretically speaking, this must be the optimum level of production for white cheese that will result in the maximum possible profit for this particular product. First of all, the cumulative probability resulting from formula (2.11), i.e.  $U_1(Q_{s_1}) \leq \frac{2P-V+1}{3P-2V+1}$ ,

$$U_1(Q_{s_1}) \leq \frac{2P-V+1}{3P-2V+1}$$

should be the same as the one in Table 1, where the probability of the demand being equal to or smaller than the production is 85%,

$$\text{Therefore, } U(Q_s) \leq \frac{2P-V+1}{3P-2V+1} \leq \frac{121}{141} \text{ or } 0.85\%$$

However, in our opinion, this result is not sufficient. It should also be verified by profits table, i.e. with formula (2.8) as follows:

$$E_1(\Pi_1 / Q\pi_1) = \left[ (P_1 - V_1) \sum_0^{Q\pi_1} Q_{s_1} P_1(Q_{s_1} / Q\pi_1) - P_1 \sum_0^{Q\pi_1} (Q\pi_1 - Q_{s_1}) P_1(Q_{s_1} / Q\pi_1) + V_1 \sum_0^{Q\pi_1} (Q\pi_1 - Q_{s_1}) P_1(Q_{s_1} / Q\pi_1) + Q\pi_1 (P_1 - V_1) \sum_{Q\pi_1+1}^{\infty} P_1(Q_{s_1} / Q\pi_1) - P_1 \sum_{Q\pi_1+1}^{\infty} (Q_{s_1} - Q\pi_1) P_1(Q_{s_1} / Q\pi_1) - FC \right]$$

The excess demand in relation to production has been estimated at 10%, which means that if the production is 500 tons, then the excess demand, if it exists based on the probabilities, will be equal to 50 tons.

Table 5 – Estimation of profits<sup>1</sup>

1.	500= 20.430,0,10 – 100,0,10.70 + 80.70,0,10 + 20.500,0,90 – 100.50,0,90 – 29000 = - 23780
2.	1500 = 20.1065,0,20 – 100,0,20.435 + 80.435,0,20 + 20.1500,0,80 – 100.150,0,80 – 29000 = -14480
3.	2000= 20.1510,0,30 – 100,0,30.490 + 80.490,0,30 + 20.2000,0,70 – 100.200,0,70 – 29000 = - 8880
4.	2500= 20.1870,0,40 – 100,0,40.630 + 80.630,0,40 + 20.2500,0,60 – 100.250,0,60 – 29000 = - 4080
5.	3000= 20.2085,0,45 – 100,0,45.915 + 80.915,0,45 + 20.3000,0,55 – 100.300,0,55 – 29000 = - 1970
6.	3500= 20.2488,0,70 – 100,0,70.1012 + 80.1012,0,70 + 20.3500,0,30 – 100.350,0,30 – 29000 = + 2164
7.	4000= 20.2920,0,80 – 100,0,80.1080 + 80.1080,0,80 + 20.4000,0,20 – 100.400,0,20 – 29000 = + 8470
8.	4500= 20.3375,0,85 – 100,0,85.1125 + 80.1125,0,85 + 20.4500,0,15 – 100.450,0,15 – 29000 = + 16000
9.	5000= 20.3500,0,90 – 100,0,90.1500 + 80.1500,0,90 + 20.5000,0,10 – 100.500,0,10 – 29000 = + 12000

As we can see in Table 5 with the profits, it is obvious that the maximum profit is realized when the level of production is 4,500 tons, a fact that is also confirmed by the table of  $\alpha'$  differences, since the  $\alpha'$  difference becomes negative when the level of production equals 4,500.

## 4. Conclusions and future research

After the above-mentioned and comprehensible application of the proposed model, we now formulate conclusions and make some recommendations for further research.

Every company requires realistic and accurate information which, in combination with its available resources on the one hand and its maximum capacity on the other hand, constitutes the necessary precondition for decision-making with regard to its financial activities. We believe that the methodology recommended in the present paper will enable multi-productive companies to determine the optimum level of production for various products in correspondence with their demand, and that this will be a positive contribution for the organization and execution of their production.

In our opinion, the use of such a possibility by a multi-productive company is both feasible and obviously important, given that if it is applied even for the first time and not occasionally, it will still provide the possibility to improve the profit maximization model, by adding or removing certain parameters of the revenue and expenses function.

We believe that the determination of the maximum expected profit function for each product, after taking into account the various parameters regarding final stock and loss of revenue, either due to stock or due to excess demand, can provide the entrepreneur with the possibility to rationally deal with his business problems.

In this paper, we have chosen not to refer to costing methods, as many others have done. We accept Full-cost as the ideal costing method, which determines the selling price on the market. As it was previously mentioned in this paper, we believe that the above-mentioned subject has already been thoroughly examined.

Furthermore, we place greater emphasis on determining the probability density function of demand, given that the whole philosophy of this paper focuses on stochastic demand, which plays a primary and definitive role.

This certainly does not mean that the proposed model or the method for achieving an optimum level or production, with the aim of maximizing the expected profit, provides a

<sup>1</sup> See Appendix

solution to all problems and claims to be a panacea for all. That would be an unrealistic statement indeed. The proposed model could however be combined with the variances or the standard deviations of the expected profits from each product, so that the interdependence of the demand for one product in relation to others could also be examined, using a relevant variance or covariance matrix. In our view, this could also be a subject for further research. Moreover, we believe in the golden rule governing the relation between cost and benefit. Thus, for the proposed model to be efficient, useful, practical and rational, the benefit it might offer to any multi-productive company must clearly and greatly outweigh the cost of implementing the whole process.

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## Appendix

Notes for Table 5

$P_1=100$	Selling price per unit
$V_1=80$	Variable cost per unit
$P_1 - V_1=20$	Profit margins
$\sum_0^{Q\pi} P_1(Q_{s_1}/Q\pi_1)=0,10$	Probability of demand being lower than or equal to production (Table 1)
$\sum_{Q\pi_1+1}^{\infty} P_1(Q_{s_1}/Q\pi_1)=0,90$	Probability of demand being higher than or equal to production
$Q_{s_1}=430$	Tons of demand according to Table 2.
$Q\pi_1=500$	Tons of production
$Q_{s_1} - Q\pi$	The excess demand in relation to production has been estimated at 10%, which means that if the production is 500 tons, then the excess demand, if it exists based on the probabilities, will be equal to 50 tons.
$FC=29000,00$ euro	Fixed cost
All the estimations for the other levels of production are made in the same way, based on the probabilities of Table 1 and on the demand from Table 3.	